Therefore, N(I) = S_3

2.7.2.2. Normalizer of a Sub Group

Definition: The normalizer N(H) of H for any sub-group H of a group g is defined as;

$$N(H) = \{x \in G \mid xH = Hx\}$$

Theorem 46: Let a be any element of G. Then two elements x, y $\in$ G give rise to the same conjugate to a if and only if they belong to the same right coset of the normalizer of a in G.

Proof: Suppose x, y$\in$G develop with respect to the same conjugate of a in G, then

$\Rightarrow$ $x^{-1}ax = y^{-1}ay$

$\Rightarrow$ $x(x^{-1}ax)y^{-1} = x(y^{-1}ay)y^{-1}$

$\Rightarrow$ $axy^{-1} = xy^{-1}a$

$\Rightarrow$ $xy^{-1} \in N(a)$

Since N(a) is a sub-group of G.

Hence $xy^{-1} \in N(a) \Leftrightarrow N(a)x = N(a)y$ $(a \in Hb \Leftrightarrow Ha = Hb)$

$\Rightarrow$ $x \in N(a)x = N(a)y (\because a \in Ha)$

and $y \in N(a)y = N(a)x$

$\Rightarrow$ $xy \in N(a)x = N(a)y$

Therefore x, y related to the same right coset of the normalizer of a in G.

On the other hand suppose that x, y $\in$G related to the same right cosets of a in G.

Suppose x, y $\in$ N(a)z, for some z $\in$ G

$\Rightarrow$ $N(a)z = N(a)x = N(a)y$

$\Rightarrow$ $N(a)x = N(a)y$

$\Rightarrow$ $xy^{-1} \in N(a)$ $(\because Ha = Hb \Leftrightarrow ab^{-1} \in H)$

$\Rightarrow$ $axy^{-1} = xy^{-1}a$

$\Rightarrow$ $x^{-1}(axy^{-1})y = x^{-1}(xy^{-1}a)y$

$\Rightarrow$ $x^{-1}ax = y^{-1}ay$

Therefore it is concluded that x, y $\in$ G develop with respect to the same conjugate of a in G.